

# Textures and Lepton Mass Matrices

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## Abstract

In view of the precise measurement of the leptonic mixing angle  $\theta_{13}$ , implications of the latest mixing data have been investigated for lepton mass matrices pertaining to Dirac neutrinos. These texture specific lepton mass matrices have been examined for their compatibility with the latest data in the cases of normal hierarchy, inverted hierarchy and degenerate scenario of neutrino masses.

The precise measurement of the neutrino mixing angle  $s_{13}$ [1] has added another dimension to our knowledge of neutrino oscillation phenomenology. Interestingly, this unexpectedly large value of  $s_{13}$  almost near the Cabibbo angle, seem to have important implications for flavor physics. This value which is neither like the other two mixing angles nor canonical has made lepton mixing pattern more complicated compared to expectations from hitherto believed underlying symmetries of the mixing matrix. The observation of non zero  $s_{13}$  value, on the one hand, restores the parallelism between quark mixing and lepton mixing, while on the other hand signifies the difference between the uniformly large mixing angles for leptons compared with the corresponding quark mixing angles.

Ever since the measurement of  $s_{13}$ , intense amount of activity has taken place in understanding phenomenology of the pattern of neutrino masses and mixings. Further, it becomes desirable to understand quark mixing and lepton mixing from similar perspectives, however, while following unified approach, one has to keep in mind the difference in the mixing patterns of the quarks and the leptons. In this context, one should note that unlike the case of quark mixings which show a hierarchical structure, the pattern of neutrino mixings do not show any explicitly hierarchy. Further, at present we have no clue about the hierarchy of neutrino masses which may be normal/inverted or may even be degenerate. Furthermore, exploring the relationship between neutrino mixing angles and the lepton mass matrices gets further complicated by the fact that at present it is not clear whether neutrinos are Dirac or Majorana particles.

In the absence of any viable theory for flavor physics, one usually resorts to phenomenological models such as texture specific mass matrices which have received a good deal of attention in the literature, for details in this regard we refer the reader to a recent review article . Texture specific mass matrices were introduced implicitly by Weinberg[2] and explicitly by Fritzsch[3]. In particular, Fritzsch-like texture specific mass matrices seem to be very helpful in understanding the pattern of quark mixings and CP violation. For details we refer the readers to [4] In view of the significant difference between the quark mixing pattern and the lepton mixing pattern, as well as in the absence of data regarding mass pattern of neutrinos one may have to resort to detailed and case by case analyses for all the neutrino mass hierarchies as well as for both Majorana and Dirac neutrinos, keeping in mind that Dirac neutrinos have not yet been ruled out by experiment . To this end it should be noted that several attempts[5] have been made in exploring the possibility of Dirac neutrinos having small masses as well as their compatibility with the supersymmetric GUTs.

In the case of Majorana neutrinos after the recent measurement of  $s_{13}$ , texture 6 and 5 zero mass matrices have been examined in detail[6]. However similar extensive attempts for Dirac neutrinos have not yet been carried out. Keeping in mind the parallelism between quark and lepton mixing phenomena and noting that the Dirac neutrinos have not yet been ruled out, it becomes desirable to study texture specific Dirac neutrino mass matrices. In this context, it may be added that the original texture 6 zero Fritzsch-like mass matrices have been ruled out in the case of quarks, similarly a closer look at some of these attempts indicate the same for texture 6 zero Dirac mass matrices[7].

In the case of quarks it has been shown that texture 5 zero mass matrices have been largely ruled out however, a detailed analysis has not been carried out for the case of Dirac neutrinos. Therefore it becomes desirable to carry out detailed analyses of texture 5 zero Dirac neutrino mass matrices, this is particularly important in view of recent refinements and measurement of angle  $s_{13}$ . The plan of present work is as follows. Firstly to make the document self contained we discuss the texture specific Dirac neutrino mass matrices in Section I. In Section II we discuss the inputs used for analyses. In Section III we present the complete analyses of texture 5 zero Dirac mass matrices for normal hierarchy, inverted hierarchy and degenerate scenario. Lastly in Section IV we summarize our conclusions.

## 1 Texture 5 zero Dirac neutrino mass matrices

To define the various texture specific cases considered here, we begin with the modified Fritzsch-like mass matrices, for example,

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & C_l \end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & C_\nu \end{pmatrix}, \quad (1)$$

$M_l$  and  $M_{\nu D}$  respectively corresponding to Dirac-like charged lepton and neutrino mass matrices. Both the matrices are texture 2 zero type with  $A_{l(\nu)} = |A_{l(\nu)}|e^{i\alpha_{l(\nu)}}$  and  $B_{l(\nu)} = |B_{l(\nu)}|e^{i\beta_{l(\nu)}}$ . The two possible cases of texture 5 zero matrices can be obtained by taking either  $D_l = 0$  and  $D_\nu \neq 0$  or  $D_\nu = 0$  and  $D_l \neq 0$ , referred to as texture 5 zero  $D_l = 0$  case pertaining to  $M_l$  texture 3 zero type and  $M_{\nu D}$  texture 2 zero type and texture 5 zero  $D_\nu = 0$  case pertaining to  $M_l$  texture 2 zero type and  $M_{\nu D}$  texture 3 zero type. The formalism connecting the mass matrix to the neutrino mixing matrix involves diagonalization of the mass matrices  $M_l$  and  $M_{\nu D}$  and the details in this regard can be looked up in [4], however to facilitate discussion of results we briefly present some of the essentials in this regard. To facilitate diagonalization, the mass matrix  $M_k$ , where  $k = l, \nu D$  can be expressed as

$$M_k = Q_k M_k^r P_k \quad (2)$$

or

$$M_k^r = Q_k^\dagger M_k P_k^\dagger, \quad (3)$$

where  $M_k^r$  is a real symmetric matrix with real eigenvalues and  $Q_k$  and  $P_k$  are diagonal phase matrices. The real matrix  $M_k^r$  is diagonalized by the orthogonal transformation  $O_k$ , for example,

$$M_k^{diag} = O_k^T M_k^r O_k, \quad (4)$$

which on using equation (3) can be rewritten as

$$M_k^{diag} = O_k^T Q_k^\dagger M_k P_k^\dagger O_k. \quad (5)$$

To understand the relationship between diagonalizing transformations for different hierarchies of neutrino masses as well as their relationship with the charged lepton case, we reproduce the general diagonalizing transformation  $O_k$ . The elements of  $O_k$  can figure with different phase possibilities, however these possibilities are related to each other through the phase matrices. For the present work, we have chosen the possibility,

$$O_k = \begin{pmatrix} O_k(11) & O_k(12) & O_k(13) \\ O_k(21) & -O_k(22) & O_k(23) \\ -O_k(31) & O_k(32) & O_k(33) \end{pmatrix}, \quad (6)$$

where,

$$\begin{aligned} O_k(11) &= \sqrt{\frac{m_2 m_3 (m_3 - m_2 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_3 - m_1)(m_1 + m_2)}} \\ O_k(12) &= \sqrt{\frac{m_1 m_3 (m_1 + m_3 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_2 + m_3)(m_2 + m_1)}} \\ O_k(13) &= \sqrt{\frac{m_1 m_2 (m_2 - m_1 + D_k)}{(m_1 - m_2 + m_3 - D_k)(m_3 + m_2)(m_3 - m_1)}} \\ O_k(21) &= \sqrt{\frac{m_1 (m_3 - m_2 - D_k)}{(m_3 - m_1)(m_1 + m_2)}} \end{aligned}$$

$$\begin{aligned}
O_k(22) &= \sqrt{\frac{m_2(m_3 + m_1 - D_k)}{(m_2 + m_3)(m_2 + m_1)}} \\
O_k(23) &= \sqrt{\frac{m_3(m_2 - m_1 + D_k)}{(m_2 + m_3)(m_3 - m_1)}} \\
O_k(31) &= \sqrt{\frac{m_1(m_2 - m_1 + D_k)(m_1 + m_3 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_1 + m_2)(m_3 - m_1)}} \\
O_k(32) &= \sqrt{\frac{m_2(D_k - m_1 + m_2)(m_3 - m_2 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_2 + m_3)(m_2 + m_1)}} \\
O_k(33) &= \sqrt{\frac{m_3(m_3 - m_2 - D_k)(m_1 + m_3 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_3 - m_1)(m_3 + m_2)}}, \quad (7)
\end{aligned}$$

$m_1, -m_2, m_3$  being the eigenvalues of  $M_k$ .

In the case of charged leptons, because of the hierarchy  $m_e \ll m_\mu \ll m_\tau$ , the mass eigenstates can be approximated respectively to the flavor eigenstates. Using the approximation,  $m_{l1} \simeq m_e$ ,  $m_{l2} \simeq m_\mu$  and  $m_{l3} \simeq m_\tau$ , the first element of the matrix  $O_l$  can be obtained from the corresponding element of equation (7) by replacing  $m_1, -m_2, m_3$  with  $m_e, -m_\mu, m_\tau$ , for example,

$$O_l(11) = \sqrt{\frac{m_\mu m_\tau (m_\tau - m_\mu - D_l)}{(m_e - m_\mu + m_\tau - D_l)(m_\tau - m_e)(m_e + m_\mu)}}. \quad (8)$$

In the case of neutrinos, for normal hierarchy of neutrino masses defined as  $m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}$  as well as for the corresponding degenerate case given by  $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3}$  equation (7) can also be used to obtain the first element of diagonalizing transformation for Dirac neutrinos. This element can be obtained from the corresponding element of equation (7) by replacing  $m_1, -m_2, m_3$  with  $m_{\nu_1}, -m_{\nu_2}, m_{\nu_3}$  and is given by

$$O_{\nu D}(11) = \sqrt{\frac{m_{\nu_2} m_{\nu_3} (m_{\nu_3} - m_{\nu_2} - D_\nu)}{(m_{\nu_1} - m_{\nu_2} + m_{\nu_3} - D_\nu)(m_{\nu_3} - m_{\nu_1})(m_{\nu_1} + m_{\nu_2})}}, \quad (9)$$

where  $m_{\nu_1}, m_{\nu_2}$  and  $m_{\nu_3}$  are neutrino masses.

In the same manner, one can obtain the elements of diagonalizing transformation for the inverted hierarchy case defined as  $m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2}$  as well as for the corresponding degenerate case given by  $m_{\nu_3} \sim m_{\nu_1} \lesssim m_{\nu_2}$ . The corresponding first element, obtained by replacing  $m_1, -m_2, m_3$  with  $m_{\nu_1}, -m_{\nu_2}, -m_{\nu_3}$  in equation (7) is given by

$$O_{\nu D}(11) = \sqrt{\frac{m_{\nu_2} m_{\nu_3} (m_{\nu_3} + m_{\nu_2} + D_\nu)}{(-m_{\nu_1} + m_{\nu_2} + m_{\nu_3} + D_\nu)(m_{\nu_3} + m_{\nu_1})(m_{\nu_1} + m_{\nu_2})}}. \quad (10)$$

The other elements of diagonalizing transformations in the case of neutrinos as well as charged leptons can similarly be found.

After the elements of diagonalizing transformations  $O_l$  and  $O_{\nu D}$  are known, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [8] can be obtained through the relation

$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D}, \quad (11)$$

where  $Q_l P_{\nu D}$ , without loss of generality, can be taken as  $(e^{i\phi_1}, 1, e^{i\phi_2})$ ,  $\phi_1$  and  $\phi_2$  being related to the phases of mass matrices and can be treated as free parameters.

## 2 Inputs used in the analysis

The present work uses results from the latest global three neutrino oscillation analysis carried out by Fogli *et al.* [9]. At  $1\sigma$  C.L. the allowed ranges of the various input parameters are

$$\Delta m_{21}^2 = (7.32 - 7.80) \times 10^{-5} \text{ eV}^2, \quad \Delta m_{23}^2 = (2.33 - 2.49) \times 10^{-3} \text{ eV}^2, \quad (12)$$

$$s_{12}^2 = (0.29 - 0.33), \quad s_{23}^2 = (0.37 - 0.41), \quad s_{13}^2 = (0.021 - 0.026), \quad (13)$$

where  $\Delta m_{ij}^2$ 's correspond to the solar and atmospheric neutrino mass square differences and  $s_{ij}$  corresponds to the sine of the mixing angle  $ij$  where  $i, j = 1, 2, 3$ . At  $3\sigma$  C.L. the allowed ranges are given as

$$\Delta m_{21}^2 = (6.99 - 8.18) \times 10^{-5} \text{ eV}^2, \quad \Delta m_{23}^2 = (2.19 - 2.62) \times 10^{-3} \text{ eV}^2, \quad (14)$$

$$s_{12}^2 = (0.26 - 0.36), \quad s_{23}^2 = (0.33 - 0.64), \quad s_{13}^2 = (0.017 - 0.031). \quad (15)$$

Based upon the information regarding neutrino masses and mixing parameters, Garcia *et al.* [10] have constructed the PMNS matrix taking into account the neutrino oscillation data. For example, the magnitudes of the elements of the PMNS matrix at  $3\sigma$  C.L. given by Garcia *et al.* [10] are

$$V_{\text{PMNS}} = \begin{pmatrix} 0.759 & - & 0.846 & 0.513 & - & 0.585 & 0.126 & - & 0.178 \\ 0.205 & - & 0.543 & 0.416 & - & 0.730 & 0.579 & - & 0.808 \\ 0.215 & - & 0.548 & 0.409 & - & 0.725 & 0.567 & - & 0.800 \end{pmatrix}. \quad (16)$$

It may be noted that lightest neutrino mass corresponds to  $m_{\nu_1}$  for the normal hierarchy case and to  $m_{\nu_3}$  for the inverted hierarchy case. The lightest neutrino mass, the phases  $\phi_1$ ,  $\phi_2$  and  $D_{l,\nu}$  have been considered as free parameters, in the normal hierarchy case the other two masses are constrained by  $\Delta m_{12}^2 = m_{\nu_2}^2 - m_{\nu_1}^2$  and  $\Delta m_{23}^2 = m_{\nu_3}^2 - m_{\nu_2}^2$  and by  $\Delta m_{23}^2 = m_{\nu_2}^2 - m_{\nu_3}^2$  in the inverted hierarchy case. The explored range for  $m_{\nu_1}$  is taken to be  $10^{-8} \text{ eV} - 10^{-1} \text{ eV}$ . In the absence of any constraint i.e., in the absence of CP violation in the leptonic sector, phases,  $\phi_1$  and  $\phi_2$  have been given full variation from 0 to  $2\pi$ . Although  $D_{l,\nu}$  are free parameters, however, they have been constrained such that diagonalizing transformations,  $O_l$  and  $O_\nu$ , always remain real, implying  $D_l < m_{l_3} - m_{l_2}$  whereas  $D_\nu < m_{\nu_3} - m_{\nu_2}$  for normal hierarchy and  $D_\nu < m_{\nu_1} - m_{\nu_3}$  for inverted hierarchy.

## 3 Results and discussion

### 3.1 Inverted hierarchy of neutrino masses

Parallel to the case of texture 6 zero Dirac neutrino mass matrices, we would like to carry out similar analyses for the two cases of texture 5 zero Dirac

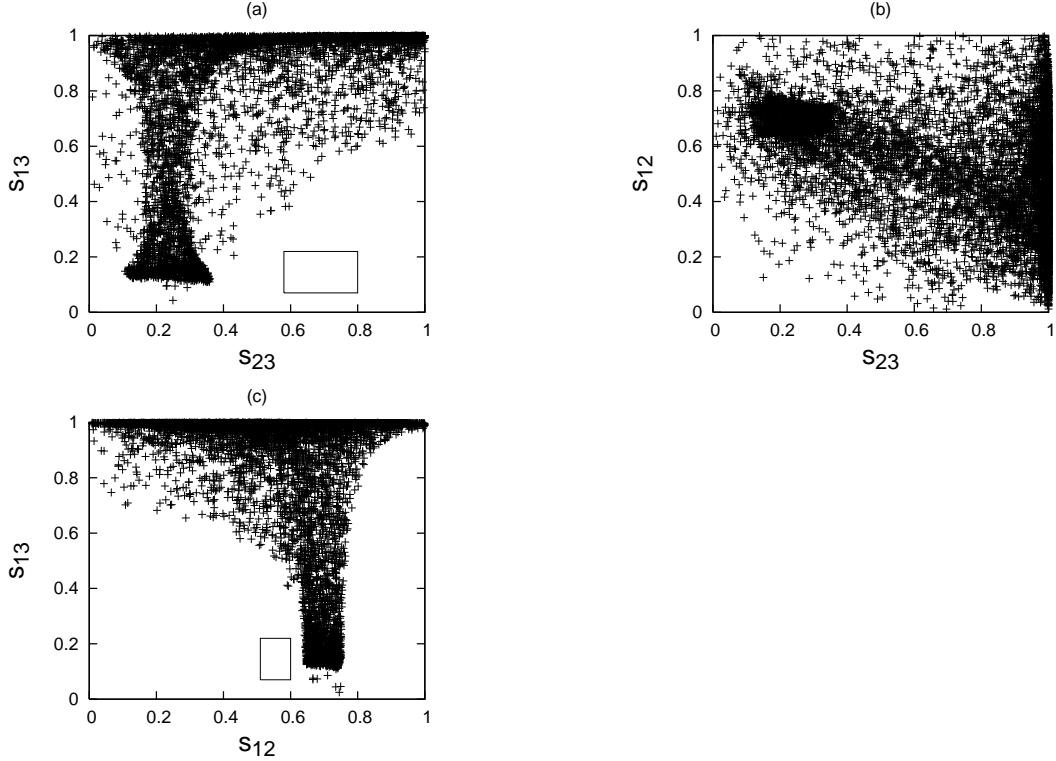


Figure 1: Plots showing the parameter space corresponding to sines of any of the two mixing angles for texture 5 zero  $D_l = 0$  case of Dirac neutrino mass matrices

neutrino mass matrices as well. As a first step, for the texture 5 zero  $D_l = 0$  case, in Figure (1) we have plotted sines of any of the two mixing angles for a particular value of  $D_\nu = m_{\nu 3}$ .

A general look at the graphs show that in the case of Figures (1a) and (1c) the blank rectangular regions show the experimentally allowed  $3\sigma$  C.L. region of the plotted angles, thus indicating towards ruling out of inverted hierarchy of neutrino masses for this case. It may also be mentioned that in Figure (1b), drawn for the sake of completion, there seems to be an absence of a blank box thereby indicating that for this case the experimentally allowed  $3\sigma$  C.L. regions of the plotted angles overlap with each other, therefore indicating towards viability of the inverted hierarchy of neutrino masses. However, it may be noted that to rule out inverted hierarchy, it is sufficient to do so from any of the three plots of Figure (1).

Coming to the next case of texture 5 zero Dirac neutrino mass matrices, i.e., the  $D_\nu = 0$  case, again in Figure (2) we have plotted sines of any of the two mixing angles. Using Figure (2c) and similar arguments as given for the previous cases, one can again conclude that this case of texture 5 zero Dirac neutrino mass matrices is ruled out for inverted hierarchy as well.

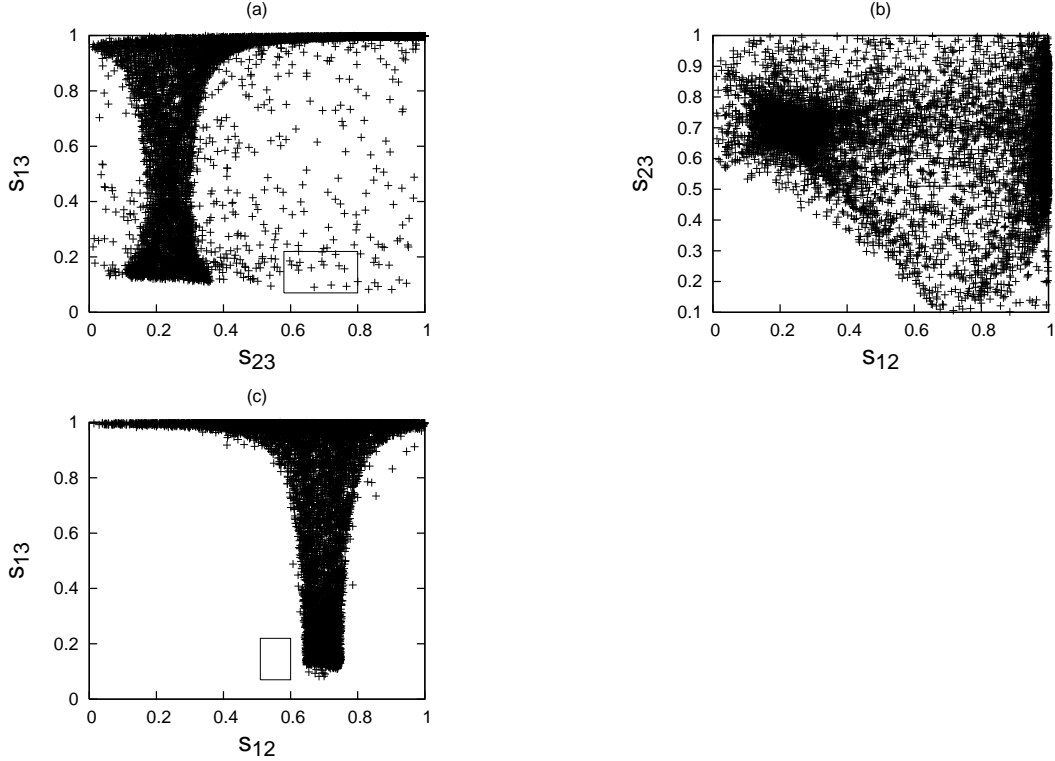


Figure 2: Plots showing the parameter space corresponding to sines of any of the two mixing angles for texture 5 zero  $D_\nu = 0$  case of Dirac neutrino mass matrices

### 3.2 Normal hierarchy of neutrino masses

For the texture 5 zero mass matrices also we would like to explore the implications of the three mixing angles on the neutrino mass  $m_{\nu_1}$ . To this end, for the  $D_l = 0$  case, in Figure (3) we present the plots of the three mixing angles versus  $m_{\nu_1}$ . The graphs shown in Figures (3b) and (3c) do not seem to provide any constraints on the neutrino mass  $m_{\nu_1}$  due to the mixing angles  $s_{13}$  and  $s_{23}$  respectively, however Figure (3b) provides an upper bound on  $m_{\nu_1} \sim 0.01\text{eV}$ .

For the sake of completion, we have also constructed the PMNS matrix for the  $D_l = 0$  case of texture 5 zero mass matrices, given as

$$U = \begin{pmatrix} 0.765 - 0.869 & 0.489 - 0.623 & 0.006 - 0.198 \\ 0.182 - 0.467 & 0.518 - 0.728 & 0.584 - 0.809 \\ 0.310 - 0.579 & 0.421 - 0.684 & 0.573 - 0.806 \end{pmatrix}. \quad (17)$$

Interestingly, the above matrix shows good deal of compatibility with a recently constructed PMNS matrix by Garcia [10]. Similarly, for the  $D_\nu = 0$  case of texture 5 zero Dirac neutrino mass matrices, in Figure (4) we have plotted the graphs showing the variation of the three neutrino mixing angles w.r.t.  $m_{\nu_1}$ . Interestingly, from a general look at the plots one finds that the  $s_{13}$  and  $s_{23}$  versus  $m_{\nu_1}$  graphs, Figures (4b) and (4c), are very similar to the corresponding plots of texture 6 zero case. It may be noted that the normal hierarchy of neutrino masses for texture 6 zero Dirac neutrino mass matrices

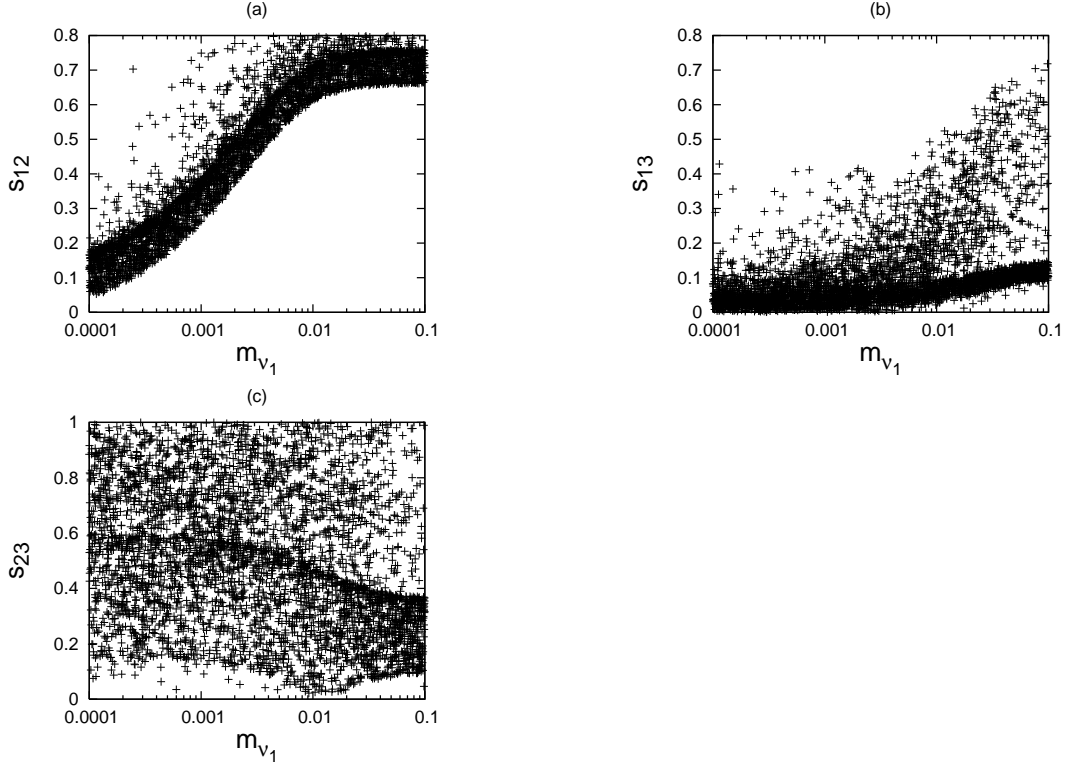


Figure 3: Plots showing the variation of the three mixing angles with the lightest neutrino mass  $m_{\nu_1}$  for the  $D_l = 0$  case of texture 5 zero Dirac neutrino mass matrices

has already been ruled out, on similar lines Figures (4b) and (4c) corresponding to the  $1\sigma$  C.L. range of  $s_{13}$ , indicate towards the ruling out of normal hierarchy for Dirac neutrinos.

Comparing the two cases of texture 5 zero Dirac neutrino mass matrices, one finds that out of the two free parameters of the mass matrices  $D_l$  and  $D_\nu$ , the parameter  $D_\nu$  plays a more important role in establishing the compatibility of texture 5 zero Dirac neutrino mass matrices. The variation of this parameter with the three mixing angles has been examined and these plots have been presented in Figure (5). A general look at these plots reveals that the mixing angles  $s_{13}$  and  $s_{12}$  seem to hardly put any restrictions on the possible values of  $D_\nu$ . However, the angle  $s_{23}$  provides a constraint on the  $D_\nu$  values, for example, from Figure (5c) one finds  $D_\nu \sim 0.01 - 0.03\text{eV}$ .

### 3.3 Degenerate scenario of neutrino masses

Coming to the case of degenerate scenario of neutrino masses for the two cases of texture 5 zero mass matrices. Parallel to the degenerate scenario for texture 6 zero mass matrices, for the  $D_\nu = 0$  case also the degenerate scenario corresponding to both normal and inverted hierarchy seems to be ruled out. This can be understood by noting that since both normal and inverted hierarchy of neutrino masses are already ruled out, therefore, the corresponding degenerate scenarios are also ruled out.



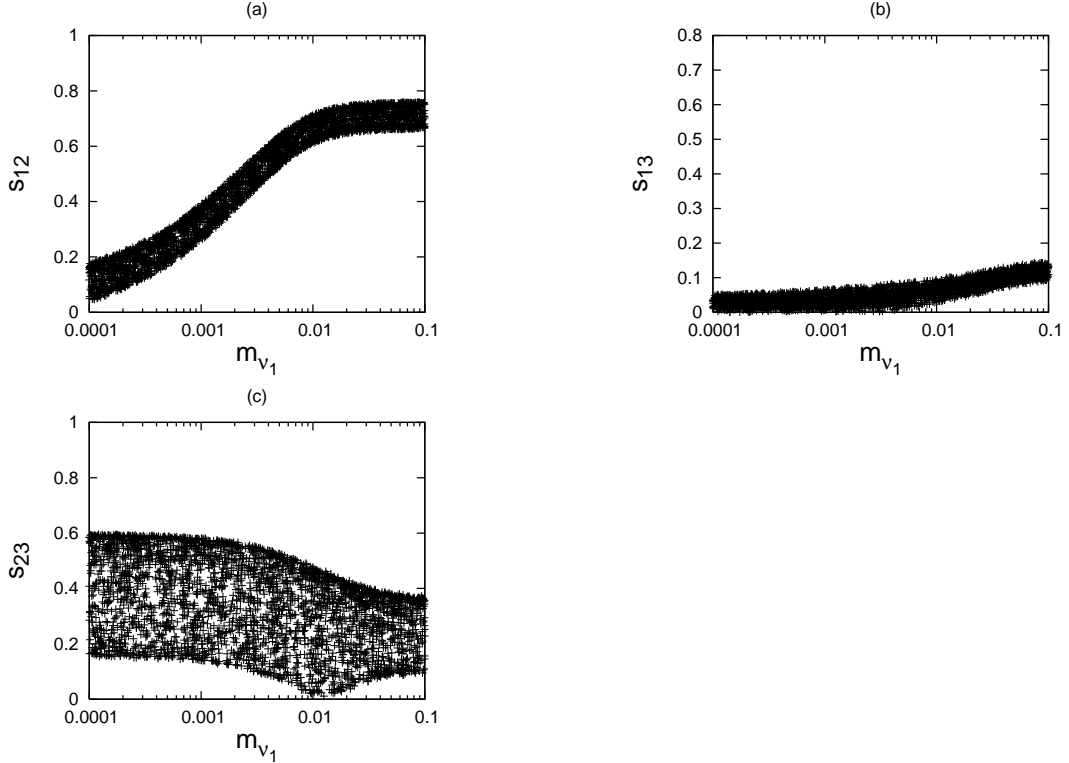


Figure 4: Plots showing the variation of the three mixing angles with the lightest neutrino mass  $m_{\nu_1}$  for the  $D_\nu = 0$  case of texture 5 zero Dirac neutrino mass matrices

For the  $D_l = 0$  case, again the degenerate scenario corresponding to inverted hierarchy is ruled out since for this case inverted hierarchy of neutrino masses is already ruled out. For the degenerate scenario following normal hierarchy of neutrino masses which is viable for this case of texture 5 zero mass matrices, again Figure (4a) can be used to rule it out. From the figure one finds that for  $m_{\nu_1}$  around 0.1 eV, there is no overlap of the plotted angle  $s_{12}$  with its experimental limits.

## 4 Summary and conclusions

To summarize, we have carried out detailed calculations pertaining to three cases, i.e., two possible cases of texture 5 zero Fritzsch-like hermitian lepton mass matrices,  $D_l = 0$  case and  $D_\nu = 0$  case. Corresponding to each of these case, we have considered three possibilities of neutrino masses having normal/inverted hierarchy and degenerate scenario. The detailed dependence of mixing angles on the lightest neutrino mass have been investigated for texture 6 zero as well as for texture 5 zero cases.

The analysis leads to several interesting results. For Dirac neutrinos, all the cases pertaining to inverted hierarchy and degenerate scenario of neutrino masses have been ruled out for texture 5 zero mass matrices. Interestingly, in the case of texture 5 zero  $D_\nu = 0$  case, the normal hierarchy of neutrino masses is also ruled out at  $1\sigma$  C.L.. Refinements in the data can make these

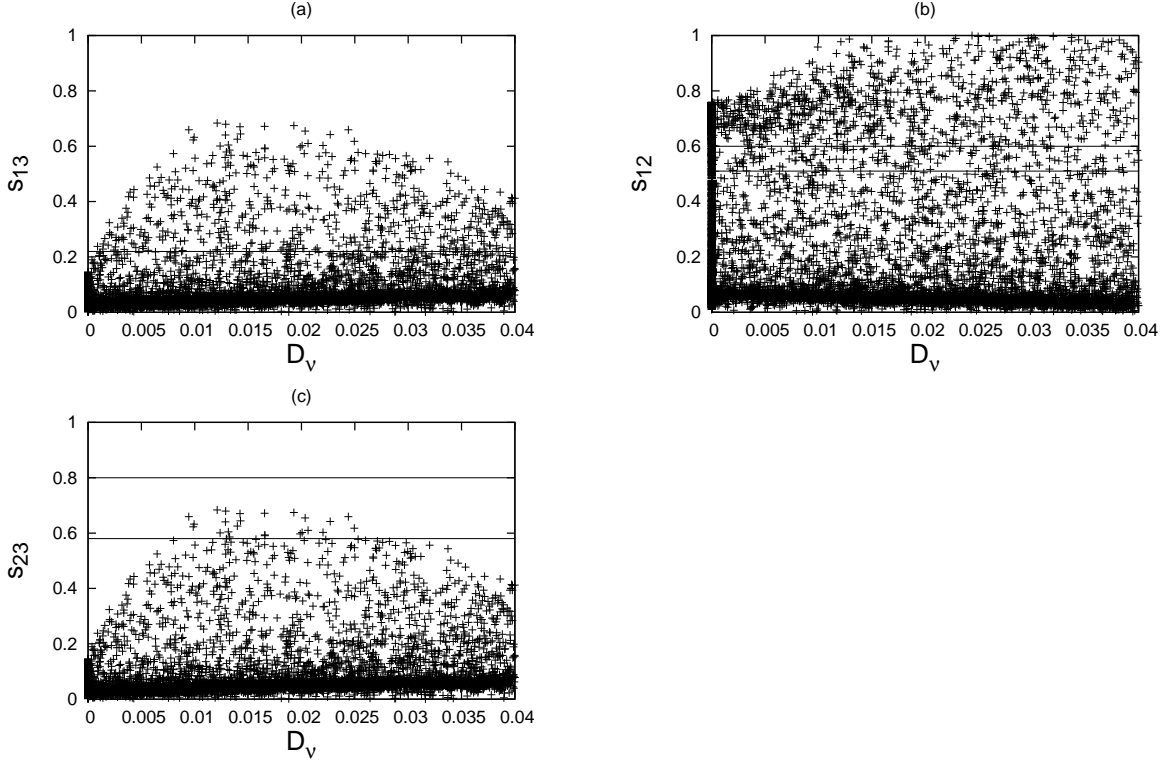


Figure 5: Plots showing variation of  $s_{13}$ ,  $s_{12}$  and  $s_{23}$  with the for texture 5 zero Dirac neutrinos for the  $D_l = 0$  case for normal hierarchy.

conclusion more rigorous.

Corresponding to the texture 5 zero  $D_l = 0$  case, the normal hierarchy of neutrino masses is viable and the plot of the mixing angle  $s_{12}$  versus  $m_{\nu_1}$  provides an upper bound on  $m_{\nu_1} \sim 0.01\text{eV}$ . The PMNS matrix for this case has also been constructed which shows good deal of compatibility with a recently constructed PMNS matrix by Garcia [10]. Further, one finds that out of the two free parameters of the mass matrices,  $D_l$  and  $D_\nu$ , the parameter  $D_\nu$  plays a more important role in establishing the compatibility of texture 5 zero Dirac neutrino mass matrices. In this context, variation of  $D_\nu$  with the three mixing angles has been examined and one finds that the angle  $s_{23}$  provides a constraint on  $D_\nu \sim 0.01 - 0.03\text{eV}$ .

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